String Field Theory as a Practical Tool for Particle Physics

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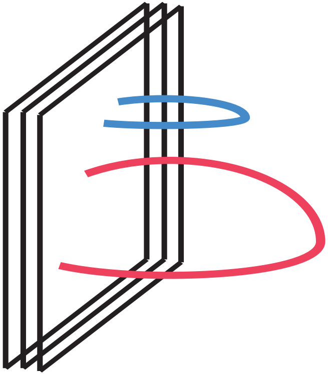
- Refs: T. Lee, (2016) arXiv:1609.01473
  T. Lee, (2017) arXiv:1701.06154

I. Introduction

- Open Bosonic String on Multiple D-branes

$$\Psi[X] \quad = \quad \frac{1}{\sqrt{2}}\Psi^0[X] + \Psi^a[X]T^a, \quad a = 1, \dots, N^2 - 1$$

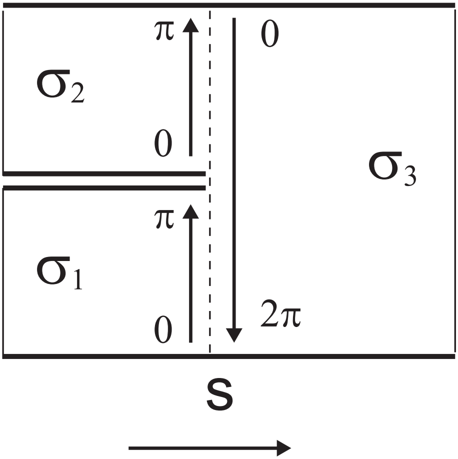
$$\text{Non-Abelian group} : U(1) \otimes SU(N)$$



- Mode expansions: With length parameters α_r

$$
\begin{aligned}
X^{(r)}(\sigma_r) &= x^{(r)} + 2\sum_{n=1} \frac{1}{\sqrt{n}} x_n^{(r)} \cos\left(n\frac{\sigma_r}{|\alpha_r|}\right), \\
&= x^{(r)} + \sum_{n=1} \frac{i}{\sqrt{n}} \left(a_n^{(r)} - a_n^{(r)\dagger}\right) \cos\left(n\frac{\sigma_r}{|\alpha_r|}\right), \\
P^{(r)}(\sigma_r) &= \frac{1}{\pi|\alpha_r|}\left(p^{(r)} + \sum_{n=1} \sqrt{n}\, p_n^{(r)} \cos\left(n\frac{\sigma_r}{|\alpha_r|}\right)\right) \\
&= \frac{1}{\pi|\alpha_r|}\left(p^{(r)} + \sum_{n=1} \sqrt{n}\left(a_n^{(r)} + a_n^{(r)\dagger}\right) \cos\left(n\frac{\sigma_r}{|\alpha_r|}\right)\right).
\end{aligned}
$$

- Three-String Vertex (String Interaction)



- String Field Action

$$S = \operatorname{tr}\int D[X]\,\Psi\,(L_0 - i\epsilon)\,\Psi$$
$$+\operatorname{tr}\int \prod_{r=1}^{3} D[X^{(r)}]\delta[X^{(1)}, X^{(2)}; X^{(3)}]\Psi[X^{(1)}]\Psi[X^{(2)}]\Psi^3[X^{(3)}].$$

Here $L_0 = \frac{p^2}{2} + N - 1$ and

$$\delta[X^{(1)}, X^{(2)}; X^{(3)}] = \prod_{\sigma} \delta\left[X^{(3)}(\sigma_3)\Theta_3 - X^{(1)}(\sigma_1)\Theta_1 - X^{(2)}(\sigma_2)\Theta_2\right]$$

$$\Theta_1 = \theta(\pi - \sigma), \quad \Theta_2 = \theta(\sigma - \pi), \quad \Theta_3 = \Theta_1 + \Theta_2 = 1.$$

II. Why String Field Theory is Difficult ?

- Overlapping function in momentum space

$$\delta[P^{(1)}, P^{(2)}; P^{(3)}] \;=\; \prod_{\sigma} \delta\Big[P^{(3)}(\sigma_3)\Theta_3 + P^{(1)}(\sigma_1)\Theta_1 + P^{(2)}(\sigma_2)\Theta_2\Big].$$

- Overlapping function in terms of normal modes

$$\begin{aligned} p^{(1)} + p^{(2)} + p^{(3)} &= 0, \\ p^{(3)}_m + \sum_{n=1}^{\infty}\left(A^{(1)}_{mn}p^{(1)}_n + A^{(2)}_{mn}p^{(2)}_n\right) + B^{(1)}_m p^{(1)} + B^{(2)}_m p^{(2)} &= 0, \quad m \neq 0 \end{aligned}$$

- Fock space representation

$$\begin{aligned} \psi_n(p) &= \langle n|p\rangle, \\ |p\rangle &= (\text{constant})\exp\left(-\frac{1}{4}p^2 + p\,a^\dagger - \frac{1}{2}a^\dagger a^\dagger\right)|0\rangle. \end{aligned}$$

Fock Space Representation of the String Vertex

- Three-String Interaction: $\langle \Psi_1, \Psi_2, \Psi_3 | \boldsymbol{V}_{[3]} \rangle$

$$
|\boldsymbol{V}_{[3]}\rangle = \int \prod_r D[P^{(r)}](2\pi)^d \delta\left(\sum_{r=1}^{3} p_r\right)
$$

$$
\exp\left[\sum_{n=1}^{\infty}\sum_{r=1}^{3}\left(-\frac{1}{4}\left(p_n^{(r)}\right)^2 + p_n^{(r)} a_n^{(r)\dagger} - \frac{1}{2}\left(a_n^{(r)\dagger}\right)^2\right)\right]|0\rangle
$$

$$
\prod_{m=1} \delta\left[p_m^{(3)} + \sum_{n=1}^{\infty}\left(A_{mn}^{(1)} p_n^{(1)} + A_{mn}^{(2)} p_n^{(2)}\right) + B_m^{(1)} p^{(1)} + B_m^{(2)} p^{(2)}\right].
$$

Performing the Gaussian integrations over the momentum modes, we may obtain the Fock space representation. However, this task is rather involved.

III. Witten's Cubic String Field Theory (1986)

- Chern-Simons Three-Form Action

$$S = \int \left(\Psi * Q\Psi + \frac{2g}{3} \Psi * \Psi * \Psi \right)$$

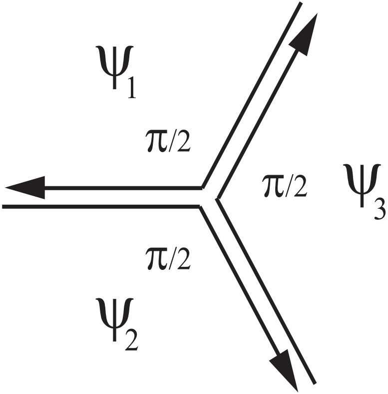
- Gauge Invariance

$$\delta\Psi = Q\Psi + \Psi * \epsilon - \epsilon * \Psi.$$

- Axioms of Algebra

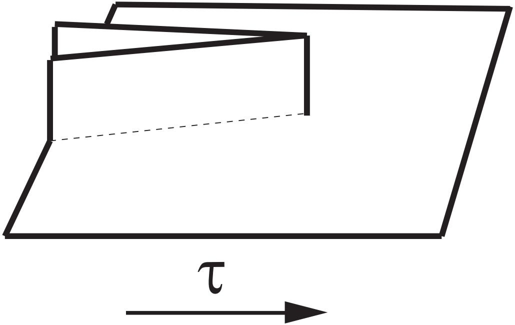
$$\begin{aligned} Q(\Psi_1 * \Psi_2) &= (Q\Psi_1) * \Psi_2 + (-1)^{n_{\Psi_1}} \Psi_1 * (Q\Psi_2), \\ (\Psi_1 * \Psi_2) * \Psi_3 &= \Psi_1 * (\Psi_2 * \Psi_3) \ \ \text{(Associativity)}. \end{aligned}$$

Mid-point overlapping interaction



World Sheet Diagram

- Mid-point overlapping interaction



- World sheet diagram of Witten's cubic string field theory is not planar. It is a conical space with excess angle π.
- It is difficult to map the world sheet diagram onto a planar space like the upper half plane or a disk.
- See, however for the three-string vertex,
  D. J. Gross and A. Jevicki, Nucl. Phys. B **287** 225 (1987) and for the four-string vertex
  S. B. Giddings, Nucl. Phys. B **278**, 242 (1986).
- Problem: Can we get the non-Abelian Yang-Mills gauge theory from the string field theory defined on multi-D-brane ?
- It is not yet clear we do have correct Fock space representations of the string vertices.

IV. New Proposal: Covariant SFT in Proper-Time Gauge

- Lee (1987, 1988, 2016, 2017)
- Line element along the spatial direction in the proper-time gauge

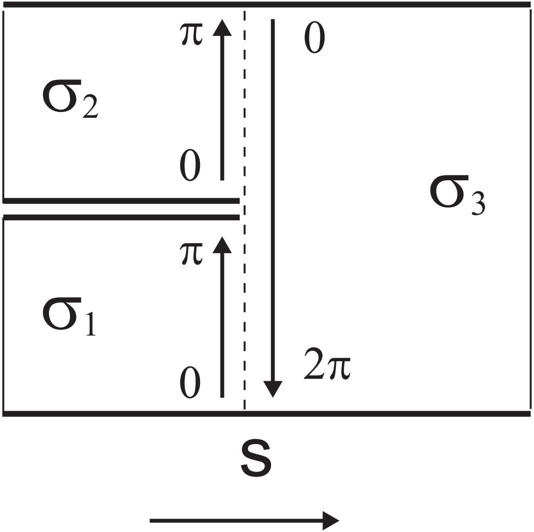
$$\Delta = \frac{1}{\sqrt{n_1}}|\sigma' - \sigma|.$$

- Choose a common proper-time for all three strings so that the energy scale would be same at the junction of three strings

$$n_1^{(1)} = n_1^{(2)} = n_1^{(3)} = n_1\,.$$

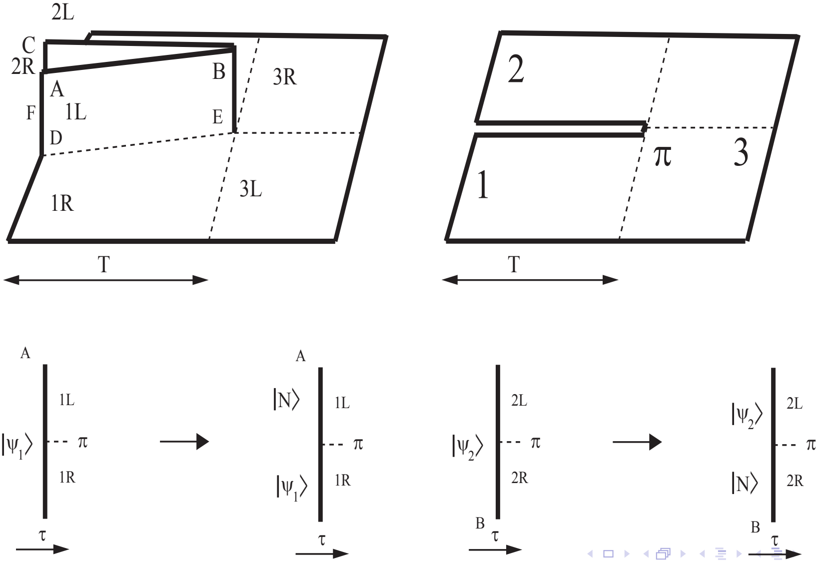
- Sum of the actual lengths of the first two open strings must be that of the third string

$$\frac{\pi}{\sqrt{n_1}} + \frac{\pi}{\sqrt{n_1}} = \frac{2\pi}{\sqrt{n_1}}.$$


π
0
σ2
0
σ3
π
σ1
0
2π
s

Deformation of Witten’s Cubic SFT

T. Lee (2017): Deformation of the initial states



SFT in Proper-Time Gauge

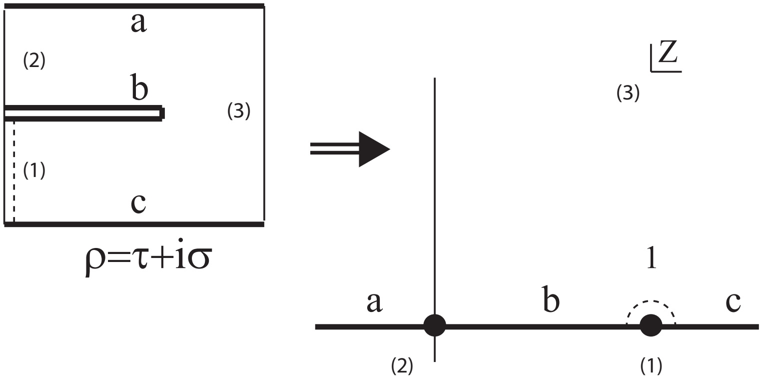
- The SFT at the proper-time gauge is a SFT with end point interaction
- It can be obtained by deforming the Witten's cubic open SFT
- It may be equivalent to fixing length parameters by choosing

$$\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = -2.$$

But it should be defined in the Fock-space representation: Otherwise the combinatorics of YM perturbation theory cannot be reproduced in the zero-slope limit. (differs from HIKKO SFT.)

Three-String Scattering Diagram

Mapping of the three-string vertex diagram onto the upper half complex plane



- Schwarz-Christoffel trnasformation (Mandelstam1973, Cremmer74, Cremmer75)

$$\rho = \ln(z-1) + \ln z.$$

- The local coordinates $\zeta_r = \xi_r + i\eta_r$, $r = 1, 2, 3$:

$$\begin{aligned} e^{-\zeta_1} &= e^{\tau_0} \frac{1}{z(z-1)}, \\ e^{-\zeta_2} &= -e^{\tau_0} \frac{1}{z(z-1)} \\ e^{-\zeta_3} &= -e^{-\frac{\tau_0}{2}} \sqrt{z(z-1)} \end{aligned}$$

V. Zero-Slope Limit

- Three-String Vertex

$$
\begin{aligned}
|V_{[3]}\rangle &= (2\pi)^d \delta\left(\sum_{r=1}^{3} p_r\right) \exp\{(E[1,2;3]\}|0\rangle, \\
E[1,2;3] &= \frac{1}{2}\sum_{n,m=1}^{\infty}\sum_{r,s=1}^{3} \bar{N}^{rs}_{nm}\alpha^{(r)}_{-n}\cdot\alpha^{(s)}_{-m} + \sum_{n=1}^{\infty}\sum_{r=1}^{3}\bar{N}^{r}_{n}\alpha^{(r)}_{-n}\cdot\mathbb{P} \\
&\quad + \sum_{r=1}^{3}\frac{\tau_0}{\alpha_r}\frac{(p^{(r)})^2}{2} - \sum_{r=1}^{3}\frac{\tau_0}{\alpha_r},
\end{aligned}
$$

where $\alpha_1 = \alpha_2 = 1$, $\alpha_3 = -2$ and $\mathbb{P} = p^{(2)} - p^{(1)}$. The momentum variable $p^{(r)}$ should be replaced by $\sqrt{\alpha'}p^{(r)}$ if we restore the slope parameter α'. Consequently, in the zero-slope limit, we may drop the 3rd term in $E[1,2;3]$; $\tau_0 \sum_{r=1}^{3} \frac{1}{\alpha_r}\frac{p_r^2}{2} \longrightarrow 0$.

Three-Gauge Field Vertex from Three-String Vertex

- External string state in the zero-slope limit

$$|\Phi^{(r)}\rangle = A^{\mu}(p_r)a_1^{(r)\dagger}{}_{\mu}|0\rangle + A^{a\mu}(p_r)T^a a_1^{(r)\dagger}{}_{\mu}|0\rangle.$$

- Neumann functions in the proper-time gauge:

$$\begin{aligned}
\bar{N}^{11}_{11} &= \frac{1}{2^4}, \quad \bar{N}^{22}_{11} = \frac{1}{2^4}, \quad \bar{N}^{33}_{11} = 2^2, \\
\bar{N}^{12}_{11} &= \bar{N}^{21}_{11} = \frac{1}{2^4}, \quad \bar{N}^{23}_{11} = \bar{N}^{32}_{11} = \frac{1}{2}, \quad \bar{N}^{31}_{11} = \bar{N}^{13}_{11} = \frac{1}{2}, \\
\bar{N}^{1}_{1} &= \bar{N}^{2}_{1} = \frac{1}{4}, \quad \bar{N}^{3}_{1} = -1, \quad \tau_0 = -2\ln 2.
\end{aligned}$$

Three-Gauge Field Vertex from Three-String Vertex

- Three-string vertex in the zero-slope limit

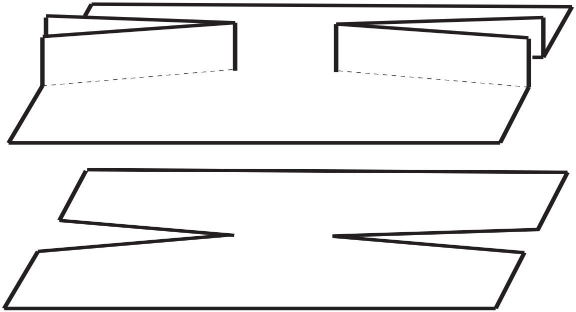
$$
\begin{aligned}
V_{[3]} &= \frac{i}{2} g_{YM} \int \prod_{r=1}^{3} \frac{d\,p^{(r)}}{(2\pi)^d} (2\pi)^d \delta \left(\sum_{r=1}^{3} p^{(r)} \right) \\
&\quad f^{abc} A^{\mu}{}_a(p_1) A^{\nu}{}_b(p_2) A^{\lambda}{}_c(p_3) \eta_{\mu\nu} (p_2 - p_1)_\lambda, \\
g_{YM} &= (\alpha')^{d/4-1} g.
\end{aligned}
$$

- The three-gauge field interaction term follows from it:

$$
V_{[3]\mathrm{Gauge}} = \frac{g_{YM}}{2} \int d^d x\, f^{abc} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu \right) A^{\mu b} A^{\nu c}.
$$

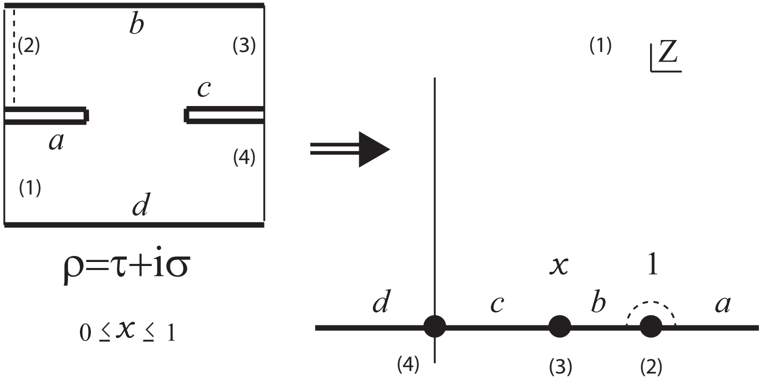
Effective Four-String Vertex of the Witten's SFT and Deformation to Planar Diagram

Length parameters: $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 = \alpha_4 = -1$.



Schwarz-Christoffel Mapping for Four-String Vertex

$$\rho = \ln(z-1) - \ln(z-x) - \ln z.$$
$$Z_1 = \infty, \quad Z_2 = 1, \quad Z_3 = x, \quad Z_4 = 0.$$



Four-Gauge Interaction

$$
\begin{aligned}
S_{[4]} \quad = \quad & \frac{1}{2} \times 2g^2_{YM} \int \prod_{r=1}^{4} dp^{(r)} \delta \left(\sum_{r=1}^{4} p^{(r)} \right) \int \left| \frac{\prod_{r=1}^{4} dZ_r}{dV_{abc}} \right| \\
& \prod_{r<s} |Z_r - Z_s|^{p_r \cdot p_s} \exp \left[- \sum_{r=1}^{4} \bar{N}^{[4]rr}_{00} \right] \mathrm{tr} \langle 0 | \left(\prod_{i=1}^{4} A(p^{(i)}) \cdot a^{(i)}_1 \right) \\
& \frac{1}{2!} \times \frac{1}{2^2} \left\{ \sum_{r,s=1}^{4} \bar{N}^{[4]rs}_{11} a^{(r)\dagger}_1 \cdot a^{(s)\dagger}_1 \right\}^2 |0\rangle .
\end{aligned}
$$

The four-string vertex is obtained from the three-string vertex by using the Cremmer-Gervais identity (1974)

Four-Gauge Interaction

- Four-Gauge term

$$
\begin{aligned}
S_{[4]} &= g_{YM}^2 \int \prod_{r=1}^{4} dp^{(r)} \delta\left(\sum_{r=1}^{4} p^{(r)}\right) \\
&\quad \mathrm{tr}\Bigg(A^\mu(p_1) A^\nu(p_2) A_\mu(p_3) A_\nu(p_4) \\
&\quad + \frac{2u}{s} A^\mu(p_1) A_\mu(p_2) A^\nu(p_3) A_\nu(p_4) \Bigg)
\end{aligned}
$$

- It contains the contact quartic gauge term and the effective four-gauge field interaction term mediated by the massless gauge field.
- Mandelstam variables:

$$
s = -(p_1 + p_2)^2, \quad t = -(p_1 + p_4)^2, \quad u = -(p_1 + p_3)^2.
$$

Four-Gauge Interaction

- Contact quartic gauge term

$$
\begin{aligned}
S_{[4]} &= S_{\text{Gauge}[4]} + S_{\text{Massless}[4]}, \\
S_{\text{Gauge}[4]} &= \frac{g_{YM}^2}{2} \int d^d x \, \mathrm{tr} \, [A^\mu, A^\nu] \, [A_\mu, A_\nu] \\
S_{\text{Massless}[4]} &= g_{YM}^2 \int \prod_{i=1}^{4} dp^{(i)} \delta \left(\sum_{i=1}^{4} p^{(i)} \right) \left(1 + \frac{2u}{s} \right) \\
& \quad \mathrm{tr} \left(A(p^{(1)}) \cdot A(p^{(2)}) A(p^{(3)}) \cdot A(p^{(4)}) \right).
\end{aligned}
$$

- The open string field theory in the proper time gauge correctly reproduces the Yang-Mills gauge field action in the zero-slope limit. This result also confirms that the Witten's cubic open string field theory reduces to the Yang-Mills gauge field in the zero-slope limit if deformed appropriately.

Conclusions

- The Witten's cubic string field theory is also shown to reduce to the $U(N)$ YM gauge field theory in the zero-slope limit by deforming their non-planar diagram into the planar diagrams of the SFT in the proper-time gauge.
- Scattering Amplitudes with an arbitrary number of external particles are now calculable.
- Applicable to scattering amplitudes for particles with higher spins
- Applications: Calculation of stringy corrections (α' corrections)
- Application in Tachyon condensation
- Extensions: BRST invariance, Super-symmetric string field theory